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A Fault Isolation Filter Design Using Left Eigenstructure Assignment Scheme

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This paper presents a novel fault isolation filter design method using left eigenstructure assignment scheme proposed by the first author et al. The proposed method shows good performance of fault isolation with an exact eigenstructure assignment and guarantees that the corrupted r faults can be isolated simultaneously when the number of available output measurements are equal to or larger than $(r+1)$. A numerical example for the fault isolation filter is also included.

Key Words : Fault Detection and Isolation, Left Eigenstructure Assignment

1. Introduction

In recent years, there has been growing interest in the application of model-based fault detection and isolation theory in the aerospace, chemical, automotive, and other industries (Park and Rizzoni, 1994). A fault occurring in these industries may cause loss of lives and huge financial damage, so the prevention of faults is becoming more important.

In the early years of research, fault detection and isolation was achieved simply through hardware replication. For example, if a required sensor is triplicated or quadruplicated, the outputs of sensors can be compared directly with each other, and if one output deviates significantly from the others, that sensor can be declared a failure (Massoumnia and Vander Velde, 1988 ; Saif and Guan, 1993). However, recently, due to the inefficiency of the replication of components, interest has shifted to the use of analytic redundancy rather than massive hardware redundancy. Unlike hardware replication, analytic redundancy depends on a dynamic model of a controlled system to relate actuator commands and sensor outputs, thus enabling checks for consistency without directly comparing the outputs of replicated components. However, this approach is known to have an undesired consequence, that is, errors in the representation of the system dynamics or parameter variations may cause confusion in the detection and isolation of component failures (Saif and Guan, 1993).

Many FDI (Fault Detection and Isolation) methods have been developed to accommodate unknown exogenous disturbances in the dynamics of considered plants. Willsky (1976) proposed several fault diagnosis methods for a given system model, and Isermann (1984) and Frank (1990) have arranged and classified the following ana-

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lytic redundancy methods: the parity space method (PSM), the parameter identification method (PIM), and the observer method (OM).

Among the above-mentioned approaches, the method using an unknown input observer scheme is widely used for FDI, since most actuator failures can be generally modeled as unknown inputs of the system. However, in the unknown input observer, fault isolation results can only be obtained under certain conditions, for example, a system must be observable, and the eigenvalues must be assigned to different values, etc. Under any condition, a bank of observers must be employed to isolate multiple faults, since one observer is designed to monitor a single fault only, with all other faults being modeled as unknown inputs (Liu and Si, 1997). In the method presented in (Patton and Chen, 1991) and (Patton and Kangethe, 1989), an eigenstructure assignment scheme is used for designing a diagnostic observer, however, a bank of observers is nevertheless still needed to accommodate multiple faults.

Liu and Si (1997) proposed a new filter design method to deal with this shortcoming. Their proposed filter design method is concerned with the problem of (asymptotically) isolating simultaneous multiple faults by a full-order observer for a continuous-time system. This fault isolation filter uses a static-state feedback decoupling method in which one can distinguish r simultaneous faults, if and only if r output measurements exist. However, this filter has $(n-r)$ invariant eigenvalues, and asymptotic fault isolation results can only be obtained when they are stable, where n represents the system order. For this reason, Liu and Si's method has limitations, in that it can only be used in some restricted systems.

In this paper, we propose a novel fault isolation filter design method using a left eigenstructure assignment scheme proposed by the first author et al. in (Choi, et al., 1995 ; 1998a ; 1998b). Compared with the existing Liu and Si's filter, the proposed filter shows good fault isolation performance as well as the following advantages : i) all the eigenvalues of the designed filter (n) can be assigned exactly to the desired ones (in Liu and Si's method, only r eigenvalues can be assigned to the desired ones), ii) the required conditions for designing the filter are alleviated. In this method, we can guarantee that the corrupted r simultaneous faults can be isolated when the number of available output measurements are equal to or larger than $(r+1)$.

2. Observer-Based Fault Isolation Filter

In this section, we need to establish the notations for fault isolation, which will be used in the following sections. Consider the following n-dimensional linear time-invariant system

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t), \; x(t_0) = x_0$$
$$y(t) = Cx(t) \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^l$, and $f \in R^r$ are the state, control, sensor output, and fault vectors, respectively. A, B, C are constant matrices with appropriate dimensions. It is assumed that (C, A) is observable. E, an $n \times r$ matrix with a full column rank, is considered to be the fault matrix, and f is a vector that consists of some unknown functions of time. $f(t)$ is identically equal to zero when the system functions properly, and deviates from zero whenever a fault occurs.

We intend to design an observer, which is also referred to as the fault isolation filter of the form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + H(y(t) - C\hat{x}(t)),$$
$$\hat{x}(t_0) = \hat{x}_0$$
$$r(t) = R(y(t) - C\hat{x}(t)) \tag{2}$$

where $r(t) \in R^r$ is a residual vector. Let $\varepsilon = x - \hat{x}$ be the error of state estimation, then r is governed by the following equations.

$$\dot{\varepsilon}(t) = (A - HC)\varepsilon(t) + Ef(t),$$
$$\varepsilon(t_0) = x_0 - \hat{x}_0$$
$$r(t) = RC\varepsilon(t) \tag{3}$$

where $\varepsilon(t_0)$ is the initial error of the state estimation. The matrices H and R of appropriate dimensions are the parameters to be determined such that the ith component of the residual r is decoupled (asymptotically) from all but the ith fault.

It is more convenient to represent the residual

$r(t)$ as follows:

$$r(t) = RCe^{(A-HC)(t-t_0)}\varepsilon(t_0) + RC\int_{t_0}^{t} e^{(A-HC)(t-\tau)}Ef(\tau)\,d\tau \quad (4)$$

where $\varepsilon(t_0)$ is assumed to be unknown. The design parameters H and R must be chosen so that the faults in the second term of the right-hand-side of Eq. (4) can be isolated. If all eigenvalues of $(A-HC)$ have negative real parts, then the effects of the initial estimation error will vanish as t approaches infinity. In that case, Eq. (4) can be rewritten briefly as follows:

$$r(t) = RC\int_{t_0}^{t} e^{(A-HC)(t-\tau)}Ef(\tau)\,d\tau \quad (5)$$

We can detect the fault occurrence from Eq. (5); however, we cannot isolate simultaneous faults. In order to isolate r simultaneous faults, $r(t)$ should be described in the following form:

$$r(t) = \int_{t_0}^{t} \begin{bmatrix} e^{\lambda_1(t-\tau)}f_1(\tau) \\ e^{\lambda_2(t-\tau)}f_2(\tau) \\ \vdots \\ e^{\lambda_r(t-\tau)}f_r(\tau) \end{bmatrix} d\tau \quad (6)$$

where $\lambda_1, \lambda_2, \cdots, \lambda_r$ are the desired eigenvalues of the observer.

In this paper, r *simultaneous faults are said to be isolated* if there exist H and R such that Eq. (6) is achieved. At that time, the filter is called *the fault isolation filter*. The objective of this paper is to determine matrices H and R such that r simultaneous faults can be isolated. To do this, we apply the left eigenstructure assignment scheme.

3. Left Eigenstructure Assignment for Fault Isolation

Based on the left eigenstructure assignment, a sufficient existence condition of the fault isolation filter is given in Subsection 3.1. The design procedure for the fault isolation filter is provided in Subsection 3.2.

3.1 Left eigenstructure assignment condition for fault isolation

Let Φ and Ψ be the right and left modal matrices of $(A-HC)$, respectively; that is, Φ and Ψ satisfy

$$(A-HC)\Phi = \Phi\Lambda \qquad \Psi^T(A-HC) = \Lambda\Psi^T \quad (7)$$

where Λ is the diagonal matrix with desired eigenvalues. Column vectors of Φ and Ψ are right eigenvectors and left eigenvectors of $(A-HC)$, respectively. Note that there exists biorthogonality between Φ and Ψ.

Theorem 1 (*Patten and Chen, 1991*) *If Φ and Ψ are the right and left modal matrices, respectively, then*

$$\Psi^T\Phi = I \quad (8)$$

Using Φ and Ψ, $r(t)$ can be represented by (Junkins and Kim, 1993)

$$r(t) = RC\Phi\int_{t_0}^{t} e^{\Lambda(t-\tau)}\Psi^T Ef(\tau)\,d\tau \quad (9)$$

It is not difficult to see that $r(t)$ can be expressed in the form of Eq. (6) if there exist H and R such that the following is satisfied:

$$\Psi^T E = \begin{bmatrix} I_r \\ --- \\ 0 \end{bmatrix}_{n\times r} \qquad RC\Phi = [I_r \mid \cdots]_{r\times n} \quad (10)$$

where values of $[\cdots]$ part are irrelevant.

The first step of the fault isolation filter by the left eigenstructure assignment is to determine Ψ. The left modal matrix Ψ should satisfy the following two conditions:

- Condition 1: Ψ should be achievable: that is, there should exist a filter gain H such that Eq. (7) is satisfied.
- Condition 2: Ψ should satisfy Eq. (10).

First, condition 1 is considered in the following theorem:

Theorem 2 *Let $\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ be a self-conjugate set of distinct complex numbers. Let S_{λ_i} be defined by*

$$S_{\lambda_i} \equiv [\lambda_i I_n - A^T \mid C^T]$$

and a compatibly partitioned matrix

$$R_{\lambda_i} \equiv \begin{bmatrix} N_{\lambda_i} \\ --- \\ M_{\lambda_i} \end{bmatrix} \quad (11)$$

where the columns of the matrix R_{λ_i} form a basis for the null space of S_{λ_i}. There exists a real $(n\times r)$ matrix H such that

$$\psi_i^T(A-HC)=\lambda_i\psi_i^T,\ i=1,2,\cdots,n$$

if and only if, for each i,

1) The left modal matrix $\Psi=[\psi_1,\cdots,\psi_n]$ is nonsingular.

2) $\psi_i=\psi_j^*$ when $\lambda_i=\lambda_j^*$

3) $\psi_i^T\in$span $\{N_{\lambda_i}\}$.

Proof: The proof is a straightforward dual argument of the right eigenvector assignment result in (Andry, et al., 1983). ■

The above theorem indicates that a closed-loop left eigenstructure assignment is constrained by the requirement that the left eigenvectors lie in a certain subspace.

Remark 1 *If $\{\lambda_i\}$ is a self-conjugate set of distinct complex numbers, then Ψ is nonsingular. Thus from Theorem 1, the right modal matrix Φ always exists and can be computed by $\Phi=(\Psi^T)^{-1}$. Thus, throughout this paper, it is assumed that $\{\lambda_i\}$ is a self-conjugate set of distinct complex numbers.*

Remark 2 *Using Theorem 2, we can check whether Ψ is achievable without computing H.*

Secondly, condition 2 is considered in the following lemma.

Lemma 1 *Let N_E be an augmented null space matrix E defined by*

$$N_E=[\mathrm{Null}(E_1^T)\ \mathrm{Null}(E_2^T)\cdots\mathrm{Null}(E_r^T)\ |\ \mathrm{Null}(E^T)_1\cdots\mathrm{Null}(E^T)_{(n-r)}]$$

where E_i denotes the block matrix of E, the ith column of which is removed, and Null(·) *denotes the null space of the matrix*(·). *If ψ_i is chosen as follows*

$$\psi_i=[\mathrm{Null}(E_i^T)]q_i,\ (i=1,\cdots,r)$$
$$\psi_i=[\mathrm{Null}(E^T)]q_i,\ (i=r+1,\cdots,n)$$

for some $q_i\neq 0$, $i=1,\cdots,n$, then

$$\Psi^TE=\begin{bmatrix}\alpha_1&0&0&0\\0&\alpha_2&0&0\\0&0&\ddots&0\\0&0&0&\alpha_r\\-&-&-&-\\0&0&0&0\\&\vdots&&\\0&0&0&0\end{bmatrix}\tag{12}$$

where $\Psi=[\psi_1,\cdots,\psi_n]$.

Proof: Consider $E^T\Psi$. From the construction of Ψ, Eq. (12) is immediate. ■

Now conditions 1 and 2, which are constraints imposed on Ψ, are considered simultaneously.

Lemma 2 *If ($\emptyset$ denotes the empty set)*

$$\begin{aligned}&\mathrm{Null}([N_{\lambda_i}|-\mathrm{Null}(E_i^T)])\neq\emptyset,\ (i=1,\cdots,r)\\&\mathrm{Null}([N_{\lambda_i}|-\mathrm{Null}(E^T)])\neq\emptyset,\\&(i=r+1,\cdots,n),\end{aligned}\tag{13}$$

then there exists Ψ, which satisfies conditions 1 and 2 simultaneously.

Proof: ψ_i satisfies conditions 1 and 2 simultaneously if there exist p_i and q_i satisfying

$$\psi_i=[N_{\lambda_i}]\cdot p_i=[\mathrm{Null}(E_i^T)]\cdot q_i,\ (i=1,\cdots,r)\tag{14}$$
$$\psi_i=[N_{\lambda_i}]\cdot p_i=[\mathrm{Null}(E^T)]\cdot q_i,\ (i=r+1,\cdots,n).$$

There exist p_i and q_i satisfying Eq. (14) if and only if

$$\begin{bmatrix}p_i\\--\\q_i\end{bmatrix}\in\mathrm{Null}([N_{\lambda_i}|-\mathrm{Null}(E_i^T)]),\ (i=1,\cdots,r)$$
$$\begin{bmatrix}p_i\\--\\q_i\end{bmatrix}\in\mathrm{Null}([N_{\lambda_i}|-\mathrm{Null}(E^T)]),\ (i=r+1,\cdots,n).\tag{15}$$

Now Eq. (13) implies that there exist p_i and q_i satisfying Eq. (15); thus Eq. (13) implies Eq. (14). ■

If Ψ satisfying conditions 1 and 2 exists, Φ can be computed from Eq. (8): recall that Ψ is nonsingular if λ_i is chosen as in Theorem 2 and thus $\Phi=(\Psi^T)^{-1}$. Moreover, from $\mathrm{rank}(C\Phi)=\mathrm{rank}(C)=l$, R satisfying Eq. (10) always exists if $l\geq r$.

Now we are ready to state a sufficient existence condition of R and H satisfying Eq. (10).

Theorem 3 *If $l\geq r+1$ (i.e., at least $r+1$ measurements are available), then r simultaneous faults can be isolated.*

Proof: From the previous arguments, it suffices to show that Eq. (13) is satisfied. Note the dimension of the following matrix:

$$\dim[N_{\lambda_i}|-\mathrm{Null}(E_i^T)])=n\times\alpha,\ (i=1,\cdots,r)$$
$$\dim[N_{\lambda_i}|-\mathrm{Null}(E^T)])=n\times\beta,\ (i=r+1,\cdots,n)$$

From the construction of the above matrices, α and β satisfy

$$\alpha \geq n-r+l+1,\ \beta \geq n-r+l.$$

Note that Eq. (13) is satisfied if $\alpha > n$ and $\beta > n$. From the assumption $(l \geq r+1)$, we have $\alpha > n$ and $\beta > n$. ■

3.2 Left eigenstructure assignment algorithm for fault isolation

The algorithm for the left eigenstructure assignment for fault isolation is summarized.

- *Step 1*: Determine the desired eigenvalues (λ_i), where the set $\{\lambda_i\}$ should be a self-conjugate set of distinct complex numbers.
- *Step 2*: Find the following matrices corresponding to the desired eigenvalues (λ_i)

$$S_{\lambda_i} \equiv [\lambda_i I_N - A^T | C^T],\quad R_{\lambda_i} \equiv \begin{bmatrix} N_{\lambda_i} \\ -- \\ M_{\lambda_i} \end{bmatrix}$$

where the columns of the matrix R_{λ_i} form a basis for the null space of S_{λ_i}.

- *Step 3*: Find the following matrices corresponding to the fault matrix E

$$N_E = [\mathrm{Null}(E_1^T)\ \mathrm{Null}(E_2^T) \cdots \mathrm{Null}(E_r^T) | \\ \mathrm{Null}(E^T)_1 \cdots \mathrm{Null}(E^T)_{(n-r)}]$$

where r denotes the number of the maximum simultaneous faults, and $E_i (i=1, \cdots, r)$ denotes the block matrix of E, the ith column of which is removed.

- *Step 4*: Construct the augmented achievable right modal matrix:

$$[N_{\lambda_1}, N_{\lambda_2}, \cdots, N_{\lambda_i}, \cdots, N_{\lambda_n}]$$

- *Step 5*: Determine the parameter vectors p_i and q_i satisfying Eq. (15).
- *Step 6*: For normalization of the diagonal parameters of the $\Psi^T E$ (i. e., normalize a_i in Eq. (12)), determine a parameter vector p_i^N:

$$p_i^N = \frac{1}{\|E^T N_{\lambda_i} p_i\|_2} p_i,\ (i=1, \cdots, r)$$
$$p_i^N = p_i,\ (i=r+1, \cdots, n)$$

- *Step 7*: Construct an achievable right modal matrix Ψ with a parameter vector p_i^N

$$\Psi = [N_{\lambda_1}, N_{\lambda_2}, \cdots, N_{\lambda_i}, \cdots, N_{\lambda_n}] P^N$$

where P^N is defined by

$$P^N = [p_1^N, p_2^N, \cdots, p_i^N, \cdots, p_n^N]$$

- *Step 8*: Compute vector chains and construct the matrix W as follows:

$$w_i = M_{\lambda_i} p_i^N,\ W = [w_1, w_2, \cdots, w_i, w_n]$$

- *Step 9*: Calculate the gain matrix H:

$$H^T = W(\Psi)^{-1} \tag{16}$$

- *Step 10*: Construct the matrix $\Phi = (\Psi^T)^{-1}$ and the residual output matrix R, so that Eq. (10) is satisfied.

Remark 3 *H in Eq. (16) is obtained by comparing the following equations:*

$$(\lambda_i I - A^T)\psi_i + C^T H^T \psi_i = 0$$
$$(\lambda_i I - A^T) N_{\lambda_i} p_i^N + C^T M_{\lambda_i} p_i^N = 0,$$

where the second equation is from Theorem 2. Equating $H^T \psi_i = M_{\lambda_i} p_i^N$, we obtain Eq. (16).

4. A Numerical Example

The system under consideration is a 4th-order linearized continuous system:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t)$$
$$= \begin{bmatrix} -9.947 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ -0.0 & -0.0 & 1.0 & 0.0 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 1 & 1 \\ 3 & 1 \end{bmatrix} f(t)$$

$$y(t) = Cx(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} x(t)$$

The eigenvalues of the open-loop system are

$$\Lambda^{open} = \{-6.8262\ \ -1.0117 \pm 1.5146i,\ -2.5498\}$$

Let the desired eigenvalues of the observer system be

$$\Lambda^d = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \{-1, -2, -3, -4\}$$

Then, the left modal matrix Ψ is calculated as follows:

$$\Psi=\begin{bmatrix}-0.2349 & 0.6037 & -0.0809 & -0.0808\\ -0.0548 & 0.0591 & -0.0824 & -0.0838\\ 0.0324 & 0.0498 & 0.1199 & 0.1183\\ 0.4373 & -0.2573 & 0.0420 & 0.0434\end{bmatrix}.$$

If Ψ is determined, then $\Phi=(\Psi^T)^{-1}$ can be determined, where $C\Phi$ is calculated as follows:

$$C\Phi=\begin{bmatrix}1.0000 & 2.0000 & -61.3430 & 61.0777\\ 2.0000 & 0.0000 & 415.5117 & -421.8213\\ 6.0000 & 2.0000 & 668.2516 & -671.5509\end{bmatrix}.$$

Now compute R so that Eq. (10) is satisfied:

$$R=\begin{bmatrix}-0.6719 & -1.1799 & 0.6719\\ 0.9352 & 0.8379 & -0.4352\end{bmatrix}.$$

The gain matrix H can be achieved from Eq. (16) and is given by

$$H=\begin{bmatrix}-12.6228 & -13.2004 & 7.3243\\ 87.2097 & 85.8014 & -44.4782\\ 46.0167 & 54.1450 & -30.6620\\ 0.9377 & -0.1558 & 0.5623\end{bmatrix}.$$

For the simulations of the system, we now consider the next three cases.

Case 1 : Fault input 1 is modeled as a soft bias fault ($f=5.0$) occurring at $t\geq 5s$.

Case 2 : Fault input 2 is modeled as a soft bias fault ($f=3.0$) occurring at $t\geq 10s$.

Case 3 : Simultaneous faults are given by

$$f_1=5.0(t\geq 5s),\ f_2=3.0(t\geq 10s).$$

The simulation results are presented in Figs. 1, 2, and 3. It can be seen that the fault isolation filter is functioning properly.

If Liu and Si's filter design method is applied to the above example, the observer system $(A-HC)$ has an unstable pole. Therefore, their method cannot be applied to this example.

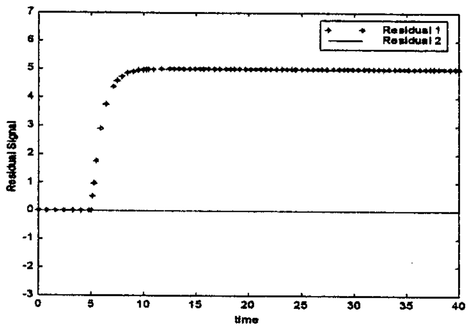

Fig. 1 (Case 1) Isolation of fault f_1

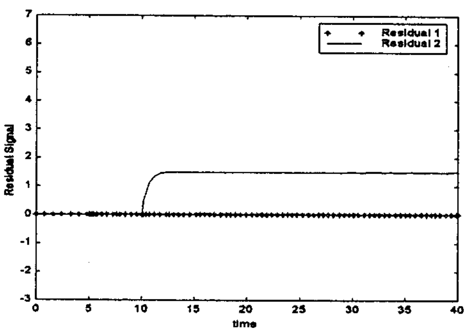

Fig. 2 (Case 2) Isolation of fault f_2

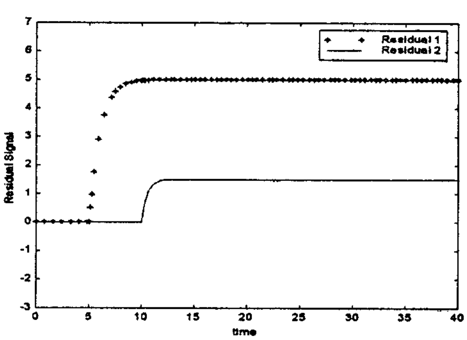

Fig. 3 (Case 3) Isolation of faults f_1 and f_2

5. Conclusion

In this paper, a fault isolation filter design methodology is proposed by using the left eigenstructure assignment scheme. For an observable system, it is shown that the corrupted r faults can be simultaneously isolated if $(r+1)$ (or more) output measurements are available. The proposed fault isolation filter design method guarantees that all the eigenvalues of a system can be assigned arbitrarily to the designed ones. The usefulness of the proposed fault isolation filter design method is verified by a numerical example.

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